CHAPTER 5

# Three Dimensional Geometry

## Recap Notes

### INTRODUCTION

Everything in the real world is in a three dimensional shape. The three dimensional geometry is used in various fields such as art and architecture, space study and astronomy, geographic information systems etc. In this chapter we shall study the direction cosines and direction ratios of line $m$ and also study about the equations of lines and planes etc.

#### Direction Cosines and Direction Ratios of a Line

- If a non-zero vector $\vec{r}$ (or any line along which $\vec{r}$ lies) makes angles $\alpha, \beta, \gamma$ with positive directions of axes, then $\alpha, \beta, \gamma$ are called direction angles of $\vec{r}$ (or of the line). $\cos\alpha, \cos\beta, \cos\gamma$ are called the direction cosines of $\vec{r}$ (or line). Any three numbers proportional to the direction cosines (d.c.'s) of a vector are called its direction ratios. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $x, y, z$ are direction ratios (numbers) and $\frac{x}{|\vec{r}|}, \frac{y}{|\vec{r}|}, \frac{z}{|\vec{r}|}$ are direction cosines of $\vec{r}$ (or any lines along which $\vec{r}$ lies).

#### Relation Between the Direction Cosines of a Line

- If $l, m, n$ are d.c.'s of a line, then $l^2 + m^2 + n^2 = 1$.

#### Direction Cosines of Line Passing Through Two Points

- Direction ratios of the line joining $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ and d.c.'s are

$$\pm\frac{x_2 - x_1}{\sqrt{\sum(x_2 - x_1)^2}}, \pm\frac{y_2 - y_1}{\sqrt{\sum(x_2 - x_1)^2}}, \pm\frac{z_2 - z_1}{\sqrt{\sum(x_2 - x_1)^2}}$$

### EQUATION OF A LINE IN SPACE

#### Equation of a Line Through a Given Point and Parallel to a Given Vector

- **Cartesian Equation :** Cartesian equation of a line passing through $(x_1, y_1, z_1)$ and having direction ratios $a, b, c$ is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

**Note :** Equation of $x$-axis, $y$-axis and $z$-axis are respectively $y = 0 = z$, $z = 0 = x$ and $x = 0 = y$.

**Vector Equation :** Vector equation of line passing through the point $A(\vec{a})$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$, where $\lambda \in R$ is a parameter.

#### Equation of a Line Passing Through Two Given Points

- **Cartesian Equation :** Cartesian equations of a line passing through two points having coordinates $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$
- **Vector Equation :** Vector equation of a line passing through two points having position vector $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, where $\lambda \in R$ is a parameter.
- **Collinearity of Three Points :** The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear, if $\frac{x_1 - x_2}{x_2 - x_3} = \frac{y_1 - y_2}{y_2 - y_3} = \frac{z_1 - z_2}{z_2 - z_3}$

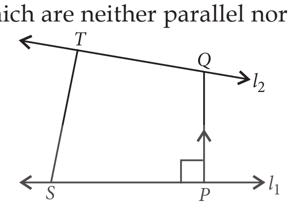
### CONDITION OF PARALLELISM AND PERPENDICULARITY OF TWO LINES

- Two lines with direction ratios $a_1, b_1, c_1$ and $a_2, b_2, c_2$ are
  (i) Perpendicular
  *i.e.*, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
  (ii) Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

### SHORTEST DISTANCE BETWEEN TWO LINES

#### Distance between Skew Lines

(The lines which are neither parallel nor intersecting.)



- Shortest distance between two skew lines is the line segment perpendicular to both the lines.
- Let $l_1$ and $l_2$ be two skew lines with equations $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$, then shortest distance in vector form is

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

If the equation of lines are given by

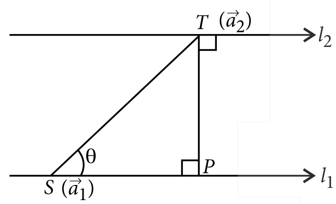
$$l_1:\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and}$$

$$l_2:\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

then shortest distance is

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

### Distance between Parallel Lines




- Let two lines $l_1$ and $l_2$ are parallel, given by $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$ respectively, where, $\vec{a}_1$ is the position vector of a point $S$ on $l_1$ and $\vec{a}_2$ is the position vector of a point $T$ on $l_2$. Then the shortest distance

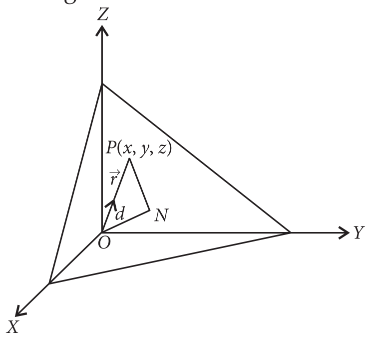
$$d=|\overrightarrow{PT}|=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

**Note :** If two lines are parallel, then they are coplanar also.

## PLANE

### Equation of a Plane in Normal Form

- **Vector Equation :** Let $\vec{r}$ be the position vector of any point $P$ on the plane. Then equation of plane is $\vec{r}\cdot\hat{n}=d$, where $\hat{n}$ is the normal unit vector to the plane and $d$ is perpendicular distance of plane from the origin.




- **Cartesian Equation :** Let $P(x, y, z)$ be any point on the plane. Then $\overrightarrow{OP}=\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

Let $l$, $m$, $n$ be the direction cosines of $\hat{n}$. Then $\hat{n}=l\hat{i}+m\hat{j}+n\hat{k}$

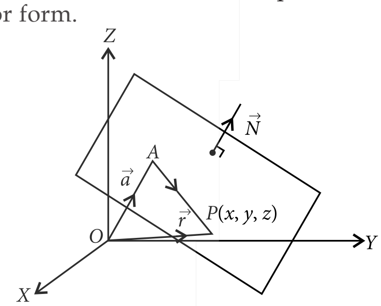
Now, $\vec{r}\cdot\hat{n}=d \quad \Rightarrow \quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(l\hat{i}+m\hat{j}+n\hat{k})=d$

$\Rightarrow \quad lx+my+nz=d$, which is cartesian equation of the plane in the normal form.

### Equation of a Plane Perpendicular to a Given Vector and Passing through a Given Point

- **Vector Equation :** Let a plane passing through a point $A$ with position vector $\vec{a}$ and perpendicular to the vector $\vec{N}$. Also, let $\vec{r}$ be the position vector of any point $P$ in the plane. Then,

$(\vec{r}-\vec{a})\cdot\vec{N}=0$, which is the equation of plane in vector form.




- **Cartesian Equation :** Let the coordinates of given point $A$ be $(x_1, y_1, z_1)$, arbitrary point $P$ be $(x, y, z)$ and direction ratios of $\vec{N}$ are $A$, $B$, and $C$. Then,

$\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}, \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

and $\vec{N}=A\hat{i}+B\hat{j}+C\hat{k}$

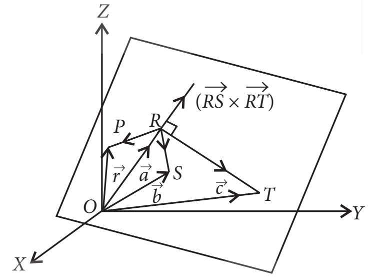
Now, $(\vec{r}-\vec{a})\cdot\vec{N}=0$

$\Rightarrow \quad A(x-x_1)+B(y-y_1)+C(z-z_1)=0$,

which is equation of plane in cartesian form.

### Equation of a Plane Passing Through Three Non Collinear Points

- **Vector Equation :** Let $R$, $S$ and $T$ be three non collinear points on the plane with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Also, let $\vec{r}$ be the position vector of any point $P$ in the plane. Then equation of plane is $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$




- **Cartesian Equation :** Let $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ be the coordinates of the points $R$, $S$ and

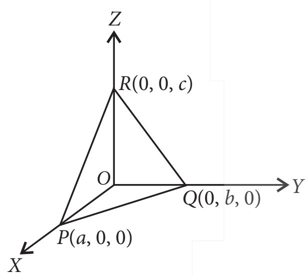
$T$ respectively. Let $(x, y, z)$ be the coordinates of any point $P$ on the plane. Then cartesian form of equation of plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

### Intercept Form of the Equation of a Plane

➤ Let plane makes intercepts $a$, $b$ and $c$ on $x$, $y$ and $z$ axes respectively. Then equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

### Equation of Plane Passing Through the Intersection of Two Given Planes




➤ **Vector Equation :** Plane passing through the intersection of the planes $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ has the equation $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2) = d_1+\lambda d_2$

➤ **Cartesian Equation :** Plane passing through the intersection of the planes $A_1x + B_1y + c_1z = d_1$ and $A_2x + B_2x + C_2z = d_2$ has the equation $(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

## COPLANARITY OF TWO LINES

➤ **Vector Form :** The lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if and only if $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 0$

➤ **Cartesian Form :** Two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar if and only if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

### Equation of Plane Containing These Two Lines

➤ **Vector Form :** Equation of the plane containing two coplanar lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, is $(\vec{r}-\vec{a}_1).(\vec{b}_1\times\vec{b}_2) = 0$

➤ **Cartesian Form :** Equation of the plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

## CONDITION OF PARALLELISM AND PERPENDICULARITY OF TWO PLANES

(i) If the planes are perpendicular, then

$\Rightarrow \quad A_1A_2 + B_1B_2 + C_1C_2 = 0$

(ii) If the planes are parallel, then $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$.

## DISTANCE OF A POINT FROM A PLANE

➤ **Vector Form :** Consider a point $P$ with position vector $\vec{a}$ and the plane $\vec{r}\cdot\vec{n} = d$.

Then the perpendicular distance from point $P$ to the plane is $\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$.

**Note :** Perpendicular distance from origin $O$ to the plane is $\frac{|d|}{|\vec{n}|}$ (since $\vec{a} = 0$).

➤ **Cartesian Form :** Let $P(x_1, y_1, z_1)$ be the given point with position vector $\vec{a}$ and $Ax + By + Cz = D$ be the cartesian equation of the given plane. Then $\vec{a} = x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$, $\vec{n} = A\hat{i}+B\hat{j}+C\hat{k}$ and the perpendicular distance from $P$ to the plane is $\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$.

➤ The distance of the point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n} = q$ measured in the direction of the unit vector $\hat{b}$ is given by $\left|\frac{q-\vec{a}\cdot\vec{n}}{\hat{b}\cdot\vec{n}}\right|$

➤ The image or reflection $(x, y, z)$ of a point $(x_1, y_1, z_1)$ in a plane $ax + by + cz + d = 0$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

➤ The foot of perpendicular $(x, y, z)$ from a point $(x_1, y_1, z_1)$ on the plane $ax + by + cz + d = 0$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

# Practice Time

## OBJECTIVE TYPE QUESTIONS

### Multiple Choice Questions (MCQs)

**1.** If a line makes an angle $\theta_1$, $\theta_2$, $\theta_3$ with the axes respectively, then the value of $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$ is

(a) 1 (b) –1
(c) 4 (d) 3

**2.** If the equation of a line $AB$ is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, find the direction ratios of a line parallel to $AB$.

(a) 1, 2, 4 (b) 1, 2, –4
(c) 1, –2, –4 (d) 1, –2, 4

**3.** If $\alpha, \beta, \gamma$ are the angles made by a line with the co-ordinate axes. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(a) 0 (b) –1 (c) 2 (d) 1

**4.** If $\alpha, \beta, \gamma$ are the direction angles of a vector and $\cos\alpha = \frac{14}{15}$, $\cos\beta = \frac{1}{3}$, then $\cos\gamma =$

(a) $\pm\frac{2}{15}$ (b) $\pm\frac{1}{5}$ (c) $\pm\frac{1}{15}$ (d) $\pm\frac{4}{15}$

**5.** If a line makes angles 90°, 60° and 30° with the positive directions of $x$, $y$ and $z$-axis respectively, then its direction cosines are

(a) $\left\langle \frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$ (b) $\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, 0 \right\rangle$

(c) $\left\langle \frac{\sqrt{3}}{2}, 0, \frac{1}{2} \right\rangle$ (d) $\left\langle 0, \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

**6.** The direction cosines of the line passing through two points (2, 1, 0) and (1, –2, 3) are

(a) $\left\langle \frac{1}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{3}{\sqrt{19}} \right\rangle$ (b) $\left\langle \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}} \right\rangle$

(c) $\left\langle \frac{-1}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}} \right\rangle$ (d) $\left\langle \frac{1}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{-3}{\sqrt{19}} \right\rangle$

**7.** Find the direction cosines of the line that makes equal angles with the three axes in space.

(a) $\pm\frac{1}{\sqrt{2}}$ (b) $\pm 1$ (c) $\pm\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

**8.** Find the direction cosines of the line joining $A(0, 7, 10)$ and $B(-1, 6, 6)$.

(a) $\left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\right)$ (b) $\left(\frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}\right)$

(c) $\left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}\right)$ (d) $\left(\frac{4}{3\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}\right)$

**9.** Find the equation of a line passing through a point (2, –1, 3) and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$.

(a) $\vec{r} = (\hat{i} + \hat{j}) + \mu(2\hat{i} - \hat{j} + 3\hat{k})$

(b) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k})$

(c) $\vec{r} = (\hat{i} - \hat{j}) + \mu(2\hat{i} - \hat{j} + 3\hat{k})$

(d) $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k})$

**10.** If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then the value of $n$ is

(a) $\frac{\sqrt{23}}{6}$ (b) $\frac{23}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

**11.** The distance of the plane $3x - 6y + 2z + 11 = 0$ from the origin is

(a) $\frac{11}{7}$ units (b) $\frac{1}{7}$ unit
(c) $\frac{7}{11}$ unit (d) $\frac{13}{7}$ units

**12.** Find the distance of the point (2, 3, 4) from the plane $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) + 11 = 0$.

(a) 1 unit (b) 4 units
(c) 2 units (d) 3 units

**13.** Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$.

(a) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$ (b) $\frac{-3}{7}, \frac{-2}{7}, \frac{6}{7}$

(c) $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$ (d) $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

**14.** If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then $k$ is equal to

(a) $-\frac{10}{7}$ (b) $-\frac{7}{10}$ (c) $-10$ (d) $-7$

**15.** The equation of a plane with intercepts 2, 3 and 4 on the $X$, $Y$ and $Z$-axes respectively is $A$. Here, $A$ refers to

(a) $2x + 3y + 4z = 12$ (b) $6x + 4y + 3z = 12$

(c) $2x + 3y + 4z = 1$ (d) $6x + 4y + 3z = 1$

**16.** What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$?

(a) 2/6 unit (b) 3/2 units

(c) 1/6 unit (d) 1/4 unit

**17.** The equation of a line is given by $\frac{4-x}{2} = \frac{y+3}{3} = \frac{z+2}{6}$, the direction cosines of line parallel to the given line is

(a) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (b) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$

(c) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (d) $\frac{-2}{7}, \frac{3}{7}, \frac{6}{7}$

**18.** A line makes angles $\alpha$, $\beta$ and $\gamma$ with the co-ordinate axes. If $\alpha + \beta = 90°$, then the value of angle $\gamma$ is

(a) 60° (b) 90° (c) 45° (d) 30°

**19.** Find the equation of a line passing through (1, 2, –3) and parallel to the line $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$.

(a) $\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$ (b) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$

(c) $\frac{x+1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$ (d) $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-1}{1}$

**20.** The vector equation of the plane passing through a point having position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ and perpendicular to the vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

(a) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = -1$

(b) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 8$

(c) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 9$

(d) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 15$

**21.** The cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. The vector equation for the line is

(a) $2\hat{i} + 3\hat{j} - 6\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 2\hat{k})$

(b) $-3\hat{i} + 5\hat{j} - 6\hat{k} + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$

(c) $-3\hat{i} - 5\hat{j} + 6\hat{k} + \lambda(2\hat{i} - 3\hat{j} - 2\hat{k})$

(d) $3\hat{i} + 5\hat{j} + 6\hat{k} + \lambda(2\hat{i} - 4\hat{j} - 2\hat{k})$

**22.** Find the equation of plane passing through the point (1, 2, 3) and the direction cosines of the normal as $l$, $m$, $n$.

(a) $lx + my + nz = l + 2m + 3n$

(b) $lx + my + nz + (l + 2m + 3n) = 0$

(c) $lx + my + nz = \frac{1}{2}(l + 2m + 3n)$

(d) None of these

**23.** The lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{k} = \frac{z+1}{7}$ are coplanar if $k =$

(a) 4 (b) –4 (c) 2 (d) –2

**24.** Find the equation of the plane passing through (2, 3, –1) and is perpendicular to the vector $3\hat{i} - 4\hat{j} + 7\hat{k}$.

(a) $3x - 4y + 7z + 13 = 0$

(b) $3x + 4y - 7z - 13 = 0$

(c) $3x + 4y + 7z - 13 = 0$

(d) $3x - 4y - 7z + 13 = 0$

**25.** The equation of a line passing through the point (– 3, 2, –4) and equally inclined to the axes are

(a) $x - 3 = y + 2 = z - 4$

(b) $x + 3 = y - 2 = z + 4$

(c) $\frac{x+3}{1} = \frac{y-2}{2} = \frac{z+4}{3}$

(d) None of these

**26.** Find the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3} = \frac{z+1}{0}$.

(a) –2, –5, 1 (b) 2, –3, 0

(c) 2, $-\frac{3}{2}$, 0 (d) $\frac{4}{5}, -\frac{3}{5}, 0$

27. The distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

(a) 1 unit (b) 7 units

(c) $\frac{1}{7}$ unit (d) 2 units

28. An equation of the plane passing through the points (3, 2, –1), (3, 4, 2) and (7, 0, 6) is $5x + 3y - 2z = \lambda$, where $\lambda$ is

(a) 23 (b) 21 (c) 19 (d) 27

29. The distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin is $A$. Here, $A$ refers to

(a) 6 (b) –6 (c) $-\frac{6}{\sqrt{29}}$ (d) $\frac{6}{\sqrt{29}}$

30. What is the distance (in units) between the two planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

(a) 0 (b) 3 (c) $\frac{6}{\sqrt{83}}$ (d) 6

31. Distance of the point $(\alpha, \beta, \gamma)$ from $y$-axis is

(a) $\beta$ (b) $|\beta|$

(c) $|\beta| + |\gamma|$ (d) $\sqrt{\alpha^2 + \gamma^2}$

32. The reflection of the point $(\alpha, \beta, \gamma)$ in the $xy$-plane is

(a) $(\alpha, \beta, 0)$ (b) $(0, 0, \gamma)$

(c) $(-\alpha, -\beta, \gamma)$ (d) $(\alpha, \beta, -\gamma)$

33. $P$ is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If $x$ co-ordinate of $P$ is 5, then its $y$ co-ordinate is

(a) 2 (b) 1 (c) –1 (d) –2

34. The equation of the line joining the points (–3, 4, 11) and (1, –2, 7) is

(a) $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z-11}{4}$

(b) $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

(c) $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+11}{4}$

(d) $\frac{x+3}{2} = \frac{y+4}{-3} = \frac{z+11}{2}$

35. The vector equation of the line through the points $A(3, 4, -7)$ and $B(1, -1, 6)$ is

(a) $\vec{r} = (3\hat{i} - 4\hat{j} - 7\hat{k}) + \lambda(\hat{i} - \hat{j} + 6\hat{k})$

(b) $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k}) + \lambda(3\hat{i} - 4\hat{j} - 7\hat{k})$

(c) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

(d) $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k}) + \lambda(4\hat{i} + 3\hat{j} - \hat{k})$

36. If the line joining (2, 3, –1) and (3, 5, –3) is perpendicular to the line joining (1, 2, 3) and $(3, 5, \lambda)$, then $\lambda =$

(a) –3 (b) 2 (c) 5 (d) 7

37. The value of $p$, so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ intersect at right angle, is

(a) $\frac{10}{11}$ (b) $\frac{70}{11}$

(c) $\frac{10}{7}$ (d) $\frac{70}{9}$

38. Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are said to be coplanar, if

(a) $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

(b) $\begin{vmatrix} x_1 & y_1 & z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$, where $(x_1, y_1, z_1)$ are the coordinates of a point on any of the line, and $a_1, b_1, c_1$ and $a_2, b_2, c_2$ are the direction ratios of $\vec{b}_1$ and $\vec{b}_2$

(c) both (a) and (b)

(d) none of these

39. The lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are

(a) coplanar (b) non-coplanar

(c) perpendicular (d) none of these

40. If the lines $\frac{x-2}{1} = \frac{y-9}{2} = \frac{z-13}{3}$ and $\frac{x-a}{1} = \frac{y-1}{-2} = \frac{z+2}{3}$ are coplanar, then $a =$

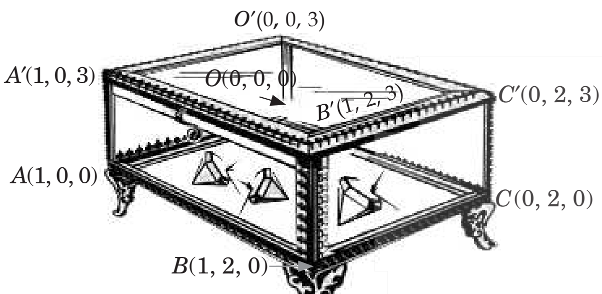
(a) 2 (b) –2

(c) 3 (d) –3

## Case Based MCQs

**Case-I :** Read the following passage and answer the questions from 41 to 45.

In a diamond exhibition, a diamond is covered in cubical glass box having coordinates $O(0, 0, 0)$, $A(1, 0, 0)$, $B(1, 2, 0)$, $C(0, 2, 0)$, $O'(0, 0, 3)$, $A'(1, 0, 3)$, $B'(1, 2, 3)$ and $C'(0, 2, 3)$.




**41.** Direction ratios of $OA$ are

(a) $< 0, 1, 0 >$ (b) $< 1, 0, 0 >$

(c) $< 0, 0, 1 >$ (d) none of these

**42.** Equation of diagonal $OB'$ is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (b) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$

(c) $\frac{x}{1} = \frac{y}{0} = \frac{z}{2}$ (d) none of these

**43.** Equation of plane $OABC$ is

(a) $x = 0$ (b) $y = 0$

(c) $z = 0$ (d) none of these

**44.** Equation of plane $O'A'B'C'$ is

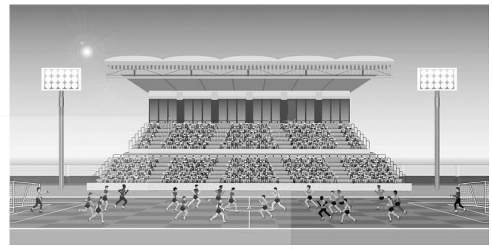
(a) $x = 3$ (b) $y = 3$ (c) $z = 3$ (d) $z = 2$

**45.** Equation of plane $ABB'A'$ is

(a) $x = 1$ (b) $y = 1$ (c) $z = 2$ (d) $x = 3$

**Case-II :** Read the following passage and answer the questions from 46 to 50.

A football match is organised between students of class XII of two schools, say school $A$ and school $B$. For which a team from each school is chosen. Remaining students of class XII of school $A$ and $B$ are respectively sitting on the plane represented by the equation $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 6$, to cheer up the team of their respective schools.



**46.** The cartesian equation of the plane on which students of school $A$ are seated is

(a) $2x - y + z = 8$ (b) $2x + y + z = 8$

(c) $x + y + 2z = 5$ (d) $x + y + z = 5$

**47.** The magnitude of the normal to the plane on which students of school $B$ are seated, is

(a) $\sqrt{5}$ (b) $\sqrt{6}$

(c) $\sqrt{3}$ (d) $\sqrt{2}$

**48.** The intercept form of the equation of the plane on which students of school $B$ are seated, is

(a) $\frac{x}{6} + \frac{y}{6} + \frac{z}{6} = 1$ (b) $\frac{x}{3} + \frac{y}{(-6)} + \frac{z}{6} = 1$

(c) $\frac{x}{3} + \frac{y}{6} + \frac{z}{6} = 1$ (d) $\frac{x}{3} + \frac{y}{6} + \frac{z}{3} = 1$

**49.** Which of the following is a student of school $B$?

(a) Mohit sitting at (1, 2, 1)

(b) Ravi sitting at (0, 1, 2)

(c) Khushi sitting at (3, 1, 1)

(d) Shewta sitting at (2, –1, 2)

**50.** The distance of the plane, on which students of school $B$ are seated, from the origin is

(a) 6 units (b) $\frac{1}{\sqrt{6}}$ units

(c) $\frac{5}{\sqrt{6}}$ units (d) $\sqrt{6}$ units

## Assertion & Reasoning Based MCQs

**Directions (Q.-51 to 60) :** In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

*(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.*

*(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.*

*(c) Assertion is correct statement but Reason is wrong statement.*

*(d) Assertion is wrong statement but Reason is correct statement.*

**51. Assertion :** The points (1, 2, 3), (–2, 3, 4) and (7, 0, 1) are collinear.

**Reason :** If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}$ and $\frac{\pi}{4}$ with $X$, $Y$, and $Z$-axes respectively, then its direction cosines are 0, $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

**52. Assertion :** If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.

**Reason :** The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

**53. Assertion :** The three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

**Reason :** The line through the points (1, –1, 2) and (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

**54. Assertion :** The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect.

**Reason :** Two lines intersect each other, if they are not parallel and shortest distance = 0.

**55. Assertion :** There exists only one plane that is perpendicular to the given vector.

**Reason :** Through a given point perpendicular to the given vector only one plane exists.

**56. Assertion :** If a variable line in two adjacent positions has direction cosines $l$, $m$, $n$ and $l + \delta l$, $m + \delta m$, $n + \delta n$, then the small angle $\delta\theta$ between the two positions is given by $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$.

**Reason :** If $O$ is the origin and $A$ is $(a, b, c)$, then the equation of plane through $A$ at right angle to $OA$ is given by $ax + by + cz = a^2 + b^2 + c^2$.

**57.** Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$.

**Assertion :** The distance of point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines $L_1$ and $L_2$ is $\frac{13}{5\sqrt{3}}$.

**Reason :** The unit vector perpendicular to both the lines $L_1$ and $L_2$ is $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

**58. Assertion :** The equation of a plane which passes through (2, –3, 1) and normal to the line joining the points (3, 4, –1) and (2, –1, 5) is given by $x + 5y - 6z + 19 = 0$.

**Reason :** The length of perpendicular from the point (7, 14, 5) to the plane $2x + 4y - z = 2$ is $2\sqrt{21}$.

**59. Assertion :** Two systems of rectangular axis have the same origin. If a plane cuts them at distances $a$, $b$, $c$ and $a'$, $b'$, $c'$ respectively from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$.

**Reason :** The points $(\hat{i} - \hat{j} + 3\hat{k})$ and $3(\hat{i} + \hat{j} + \hat{k})$ are equidistant from the plane $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$.

**60. Assertion :** The straight line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{13}$ lies in the plane $5x - y + z = 8$.

**Reason :** The straight line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{c}$ lies in the plane $ax + by + cz + d = 0$ iff normal to the plane is perpendicular to the line & every point of the line satisfies the equation of the plane.

## SUBJECTIVE TYPE QUESTIONS

### Very Short Answer Type Questions (VSA)

1. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

2. Find the vector equation of a line which passes through the points (3, 4, –7) and (1, –1, 6).

3. The equation of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

4. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the $x$-axis.

**5.** Write the vector equation of a line passing through the point (1, –1, 2) and parallel to the line whose equation is $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$.

**6.** Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.

**7.** Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

**8.** Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

**9.** Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

**10.** Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$.

## Short Answer Type Questions (SA-I)

**11.** The $x$-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its $z$-coordinate.

**12.** Find the value of $k$ so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

**13.** Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

**14.** Find the coordinates of the point where the line through (–1, 1, –8) and (5, –2, 10) crosses the $ZX$-plane.

**15.** Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

**16.** Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

**17.** Find the value of $\lambda$, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

**18.** Find the equation of the line passing through the point (–1, 3, –2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

**19.** Find the distance between the point (–1, –5, –10) and the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$.

**20.** A plane makes intercepts –6, 3, 4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it.

## Short Answer Type Questions (SA-II)

**21.** Prove that the line through $A(0, -1, -1)$ and $B(4, 5,1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$.

**22.** Show that the lines

$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$;

$\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$ are intersecting.

Hence find their point of intersection.

**23.** Using vectors, show that the points $A(-2, 3, 5)$, $B(7, 0, -1)$, $C(-3, -2, -5)$ and $D(3, 4, 7)$ are such that $AB$ and $CD$ intersect at the point $P(1, 2, 3)$.

**24.** Find the vector and cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines

$\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and

$\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

**25.** Find the shortest distance between the two lines whose vector equations are

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and

$\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

**26.** Find the direction cosines of the line $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$. Also, find the vector equation of the line through the point $A(-1, 2, 3)$ and parallel to the given line.

**27.** Find the shortest distance between the following lines :

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

**28.** Find the shortest distance between the following lines:

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

**29.** Find the unit vector perpendicular to the plane $ABC$ where the position vectors of $A$, $B$ and $C$ are $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

**30.** Find the vector equation of the plane through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$.

**31.** Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ are coplanar.

**32.** Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

**33.** Find the equation of the plane passing through the points (–1, 2, 0), (2, 2, –1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

**34.** Find the equation of a plane which passes through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

**35.** Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the $XY$-plane.

## Long Answer Type Questions (LA)

**36.** Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line, $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.
Also find the image of $P$ in this line.

**37.** Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$.

**38.** Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

**39.** Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect.
Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

**40.** Find the vector and cartesian equations of a plane containing the two lines

$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and

$\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$

Also show that the line

$\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + p(3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane.

## ANSWERS

### OBJECTIVE TYPE QUESTIONS

**1.** **(b) :** Consider, $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$
$= 2(\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3) - 3$ $\quad (\because \cos 2x = 2\cos^2 x - 1)$
$= 2(1) - 3 = -1$

**2.** **(d) :** The direction ratios of line parallel to $AB$ is 1, –2 and 4.

**3.** **(c) :** $\because$ $\alpha$, $\beta$ and $\gamma$ are the angles made by line with the co-ordinate axes.
$\therefore \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow \quad 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

**4.** **(a) :** $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$(\because \alpha, \beta, \gamma$ are direction angles$)$

$\Rightarrow \quad \frac{196}{225} + \frac{1}{9} + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = 1 - \frac{221}{225} = \frac{4}{225} \Rightarrow \cos\gamma = \pm\frac{2}{15}$

**5.** **(d) :** Let the direction cosines of the line be $l, m, n$.
Then, $l = \cos 90° = 0$, $m = \cos 60° = \frac{1}{2}$ and $n = \cos 30° = \frac{\sqrt{3}}{2}$.

So, direction cosines are $\left\langle 0, \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$.

**6.** **(c) :** Here, $P(2, 1, 0)$ and $Q(1, -2, 3)$

So, $PQ = \sqrt{(1-2)^2 + (-2-1)^2 + (3-0)^2}$

$= \sqrt{1+9+9} = \sqrt{19}$

Thus, the direction cosines of the line joining two points are $\left\langle \frac{1-2}{\sqrt{19}}, \frac{-2-1}{\sqrt{19}}, \frac{3-0}{\sqrt{19}} \right\rangle = \left\langle \frac{-1}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}} \right\rangle$

**7. (c) :** Since $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad l = m = n = \pm\frac{1}{\sqrt{3}}.$

**8. (a) :** Direction ratios of $AB$ are

$(-1 - 0, 6 - 7, 6 - 10)$ or $(-1, -1, -4)$

Also, $\sqrt{(-1)^2 + (-1)^2 + (-4)^2} = 3\sqrt{2}$

$\therefore$ Direction cosines are $\left(-\frac{1}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}\right)$

or $\left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\right)$

**9. (b) :** The given line is parallel to the vector $2\hat{i} + \hat{j} - 2\hat{k}$ and the required line is parallel to the given line. So, required line is parallel to the vector $2\hat{i} + \hat{j} - 2\hat{k}$. Thus, the equation of the required line passing through $(2, -1, 3)$ is

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k})$

**10. (a) :** $\because \left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line

$\therefore \quad \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1 \Rightarrow n^2 = \frac{23}{36} \Rightarrow n = \frac{\pm\sqrt{23}}{6}$

**11. (a) :** We have, equation of plane is $3x - 6y + 2z + 11 = 0$.
Its distance from origin $(0, 0, 0)$ is

$\left|\frac{3 \times 0 - 6 \times 0 + 2 \times 0 + 11}{\sqrt{3^2 + (-6)^2 + (2)^2}}\right| = \frac{11}{\sqrt{9 + 36 + 4}} = \frac{11}{7}$ units.

**12. (a) :** Here, $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

The distance of the point $(2\hat{i} + 3\hat{j} + 4\hat{k})$ is

$\left|\frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) + 11}{\sqrt{9 + 36 + 4}}\right| = \left|\frac{6 - 18 + 8 + 11}{7}\right| = 1$ unit

**13. (b) :** We have, $\frac{x-3}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$

$\Rightarrow$ Direction ratios are $-3, -2, 6$.

$\therefore$ Direction cosines are $\frac{-3}{7}, \frac{-2}{7}, \frac{6}{7}$.

These are direction cosines of a line parallel to given line.

**14. (a) :** Lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\Rightarrow \quad -3(3k) + 2k + 2(-5) = 0 \Rightarrow \quad k = -\frac{10}{7}$

**15. (b) :** As the plane has intercepts 2, 3 and 4 on $X$, $Y$ and $Z$ axes respectively.

$\therefore$ The required equation of the plane is

$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow 6x + 4y + 3z = 12$

**16. (c) :** Given planes are $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.

Let point $P(x_1, y_1, z_1)$ lie on plane $2x + 2y - z + 2 = 0$.

$\Rightarrow \quad 2x_1 + 2y_1 - z_1 = -2$

$\therefore \quad d = \left|\frac{4x_1 + 4y_1 - 2z_1 + 5}{\sqrt{4^2 + 4^2 + (-2)^2}}\right| = \left|\frac{2(2x_1 + 2y_1 - z_1) + 5}{\sqrt{16 + 16 + 4}}\right|$

$= \left|\frac{2(-2) + 5}{\sqrt{36}}\right| = \frac{1}{6}$ unit

**17. (d) :** Equation of given line is $\frac{4-x}{2} = \frac{y+3}{3} = \frac{z+2}{6}$.

The direction ratios of the given line are $-2, 3, 6$.

$\therefore$ The direction cosines of the given line are

$\left(\frac{-2}{\sqrt{4 + 9 + 36}}, \frac{3}{\sqrt{4 + 9 + 36}}, \frac{6}{\sqrt{4 + 9 + 36}}\right)$

$= \left(\frac{-2}{\sqrt{49}}, \frac{3}{\sqrt{49}}, \frac{6}{\sqrt{49}}\right) = \left(\frac{-2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

**18. (b) :** We know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\alpha + \cos^2(90° - \alpha) + \cos^2\gamma = 1 \quad [\because \alpha + \beta = 90°]$

$\Rightarrow \quad \cos^2\alpha + \sin^2\alpha + \cos^2\gamma = 1 \Rightarrow 1 + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = 0$

$\Rightarrow \quad \cos\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2} = 90°.$

**19. (b) :** Since, the line is parallel to the line $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$.

$\therefore$ D.r.'s of the required line are $< 1, 3, 4 >$

Hence, equation of the line passing through $(1, 2, -3)$ with d.r.'s $<1, 3, 4>$ is $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$

**20. (a) :** Vector equation of plane passing through a point having position vector $\vec{a}$ and perpendicular to $\vec{n}$ is given by $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

Here $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\therefore$ Required equation $= \vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 4 + 3 - 8 = -1$

**21. (b) :** The given cartesian equation is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$.

The line passes through the point $(-3, 5, -6)$ and is parallel to vector $2\hat{i} + 4\hat{j} + 2\hat{k}$.

Hence, the vector equation of the line is

$\vec{r} = -3\hat{i} + 5\hat{j} - 6\hat{k} + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$.

**22. (a) :** Equation of plane passing through $(1, 2, 3)$ having direction cosines of normal as $l, m, n$ is

$l(x - 1) + m(y - 2) + n(z - 3) = 0$

$\Rightarrow \quad lx + my + nz = l + 2m + 3n$

**23.** **(b) :** Given lines are $\frac{x-1}{2}=\frac{y-(-1)}{-3}=\frac{z-(-10)}{8}$

and $\frac{x-4}{1}=\frac{y-(-3)}{k}=\frac{z-(-1)}{7}$

Since the lines are coplanar.

$$\therefore \begin{vmatrix} 4-1 & -3-(-1) & -1-(-10) \\ 2 & -3 & 8 \\ 1 & k & 7 \end{vmatrix}=0$$

$\Rightarrow$ 3(– 21 – 8$k$) + 2(14 – 8) + 9(2$k$ + 3) = 0

$\Rightarrow$ 6$k$ = –24 $\Rightarrow$ $k$ = – 4

**24.** **(a) :** The equation of the plane passing through (2, 3, – 1) and perpendicular to the vector $3\hat{i}-4\hat{j}+7\hat{i}$ is

3($x$ – 2) + (– 4) ($y$ – 3) + 7($z$ – (– 1)) = 0

$\Rightarrow$ 3$x$ – 4$y$ + 7$z$ + 13 = 0

**25.** **(b) :** Since, line equally inclined to the axes.

$\therefore$ $l = m = n$ …(i)

The required equation of line is

$\frac{x+3}{l}=\frac{y-2}{l}=\frac{z+4}{l}$ [using (i)]

$\Rightarrow \frac{x+3}{1}=\frac{y-2}{1}=\frac{z+4}{1}$ $\Rightarrow$ $x$ + 3 = $y$ – 2 = $z$ + 4

**26.** **(d) :** The given line is

$\frac{x-2}{2}=\frac{2y-5}{-3}=\frac{z+1}{0}$

$\Rightarrow \frac{x-2}{2}=\frac{y-5/2}{-3/2}=\frac{z+1}{0}$

Direction cosines are

$$\frac{2}{\sqrt{2^2+\left(\frac{-3}{2}\right)^2+0^2}},\frac{-3/2}{\sqrt{2^2+\left(\frac{-3}{2}\right)^2+0^2}},\frac{0}{\sqrt{2^2+\left(\frac{-3}{2}\right)^2+0^2}}$$

*i.e.*, $\frac{2}{5/2},\frac{-3/2}{5/2},0$ *i.e.*, $\frac{4}{5},\frac{-3}{5},0$

**27.** **(a) :** We have, $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$

Here $\vec{n}=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$

$$|\vec{n}|=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{-6}{7}\right)^2}=\sqrt{\frac{49}{49}}=1$$

$\therefore$ Required distance = $\frac{|1|}{|\vec{n}|}=\frac{1}{1}=1$ unit

**28.** **(a) :** Equation of the plane through (3, 2, –1), (3, 4, 2) and (7, 0, 6) is

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ 0 & 2 & 3 \\ 4 & -2 & 7 \end{vmatrix}=0$$

*i.e.*, 5$x$ + 3$y$ – 2$z$ = 23

$\therefore$ $\lambda$ = 23

**29.** **(d) :** Distance of plane 2$x$ – 3$y$ + 4$z$ – 6 = 0 from the origin is given by,

$$\left|\frac{-6}{\sqrt{(2)^2+(-3)^2+(4)^2}}\right|=\frac{6}{\sqrt{29}}$$

**30.** **(a) :** Given planes are 3$x$ + 5$y$ + 7$z$ = 3 and 9$x$ + 15$y$ + 21$z$ = 9

$\Rightarrow$ 3$x$ + 5$y$ + 7$z$ = 3

Both the planes are coincident planes, so distance between them is zero.

**31.** **(d) :** Foot of perpendicular from ($\alpha$, $\beta$, $\gamma$) on the $y$-axis is (0, $\beta$, 0)

$\therefore$ Distance of ($\alpha$, $\beta$, $\gamma$) from $y$-axis = distance of ($\alpha$, $\beta$, $\gamma$) from (0, $\beta$, 0)

$=\sqrt{(0-\alpha)^2+(\beta-\beta)^2+(0-\gamma)^2}=\sqrt{\alpha^2+\gamma^2}$

**32.** **(d) :** Projection of $P$($\alpha$, $\beta$, $\gamma$) on $xy$-plane is $Q$($\alpha$, $\beta$, 0).

If $P'$ ($\alpha'$, $\beta'$, $\gamma'$) is the reflection of $P$ in $xy$-plane, then $Q$ is the mid-point of $PP'$.

$\Rightarrow (\alpha,\beta,0)=\left(\frac{\alpha+\alpha'}{2},\frac{\beta+\beta'}{2},\frac{\gamma+\gamma'}{2}\right)$

$\Rightarrow \frac{\alpha+\alpha'}{2}=\alpha,\frac{\beta+\beta'}{2}=\beta,\frac{\gamma+\gamma'}{2}=0$

$\Rightarrow$ $\alpha' = \alpha$, $\beta' = \beta$, $\gamma' = -\gamma$

$\therefore$ Required reflection is ($\alpha$, $\beta$, $-\gamma$).

**33.** **(a) :** Equation of line joining the points (3, 2, –1) and (6, 2, –2) is,

$\frac{x-3}{6-3}=\frac{y-2}{2-2}=\frac{z+1}{-2+1}$ *i.e.*, $\frac{x-3}{3}=\frac{y-2}{0}=\frac{z+1}{-1}=\lambda$ (say)

$\Rightarrow$ $x = 3\lambda + 3$, $y$ = 2, $z = -\lambda - 1$

So, $y$-coordinate of $P$ is 2.

**34.** **(b) :** DR's of the line joining the given points are {1 – (– 3), – 2 – 4, 7 – 11}

*i.e.*, (4, – 6, – 4) or (– 2, 3, 2)

Now, Equation of line passing through (–3, 4, 11) and having direction ratios –2, 3, 2 is $\frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$

**35.** **(c) :** If $\vec{a}=$ P.V. of $A=3\hat{i}+4\hat{j}-7\hat{k}$

and $\vec{b}=$ P.V. of $B=\hat{i}-\hat{j}+6\hat{k}$, then the equation of line $AB$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

$\therefore$ $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$

**36.** **(d) :** DR's of the given lines are 1, 2, –2 and 2, 3, $\lambda$ – 3.

Since, lines are perpendicular.

$\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow$ 1 × 2 + 2 × 3 – 2 ($\lambda$ – 3) = 0 $\Rightarrow$ $\lambda$ = 7

**37.** **(b) :** Equation of the given lines can be written in the standard form as

$$\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2} \text{ and } \frac{x-1}{-\frac{3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$$

$\because$ Lines are perpendicular to each other.

$\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0.$

$\Rightarrow$ $(-3)\left(\frac{-3p}{7}\right)+\left(\frac{2p}{7}\right)(1)+(2)(-5)=0 \Rightarrow p=\frac{70}{11}$

**38. (a) :** Two lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are coplanar if $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=0.$

**39. (a) :** Here, $x_1 = -3, y_1 = 1, z_1 = 5;$
$a_1 = -3, b_1 = 1, c_1 = 5;$
$x_2 = -1, y_2 = 2, z_2 = 5$ and $a_2 = -1, b_2 = 2, c_2 = 5$

Now, $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= 2(5-10) - 1(-15+5) + 0 = -10 + 10 = 0$
Therefore, lines are coplanar.

**40. (d) :** $\because$ The lines $\frac{x-2}{1}=\frac{y-9}{2}=\frac{z-13}{3}$ and $\frac{x-a}{1}=\frac{y-1}{-2}=\frac{z+2}{3}$ are coplanar.

$\therefore$ $\begin{vmatrix} 2-a & 8 & 15 \\ 1 & 2 & 3 \\ 1 & -2 & 3 \end{vmatrix} = 0 \Rightarrow a = -3$

**41. (b) :** D.R.'s of $OA$ are $< 1-0, 0-0, 0-0 >$, *i.e.*, $< 1, 0, 0 >$.

**42. (a) :** Equation of diagonal $OB'$ is
$\frac{x-0}{1}=\frac{y-0}{2}=\frac{z-0}{3}$ *i.e.*, $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

**43. (c) :** $OABC$ is $xy$-plane, therefore its equation is $z = 0$.

**44. (c) :** Plane $O'A'B'C'$ is parallel to $xy$-plane passing through (0, 0, 3), therefore its equation is $z = 3$.

**45. (a) :** Plane $ABB'A'$ is parallel to $yz$-plane passing through (1, 0, 0), therefore its equation is $x = 1$.

**46. (c) :** Clearly, the plane for students of school $A$ is $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=5,$ which can be rewritten as

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}+2\hat{k})=5$

$\Rightarrow$ $x + y + 2z = 5$, which is the required cartesian equation.

**47. (b) :** Clearly, the equation of plane for students of school $B$ is $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=6,$ which is of the form $\vec{r}\cdot\vec{n}=d$

$\therefore$ Normal vector to the plane is, $\vec{n}=2\hat{i}-\hat{j}+\hat{k}$ and its magnitude is $|\vec{n}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}$

**48. (b) :** The cartesian form is $2x - y + z = 6$, which can be rewritten as

$\frac{2x}{6}-\frac{y}{6}+\frac{z}{6}=1 \Rightarrow \frac{x}{3}+\frac{y}{(-6)}+\frac{z}{6}=1$

**49. (c) :** Since, only the point (3, 1, 1) satisfy the equation of plane representing seating position of students of school $B$, therefore Khushi is the student of school $B$.

**50. (d) :** Equation of plane representing students of school $B$ is $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=6,$ which is not in normal form, as $|\vec{n}|\neq 1$.

On dividing both sides by $\sqrt{2^2+(-1)^2+1^2}=\sqrt{6},$ we get

$\vec{r}\cdot\left(\frac{2}{\sqrt{6}}\hat{i}-\frac{1}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}\right)=\frac{6}{\sqrt{6}},$

which is of the form $\vec{r}\cdot\hat{n}=d$

Thus, the required distance is $\sqrt{6}$ units.

**51. (b) :** We have, $x_1 = 1, y_1 = 2, z_1 = 3;$
$x_2 = -2, y_2 = 3, z_2 = 4$ and $x_3 = 7, y_3 = 0, z_3 = 1$

Now, $\frac{x_2-x_1}{x_3-x_2}=\frac{y_2-y_1}{y_3-y_2}=\frac{z_2-z_1}{z_3-z_2}$

$\Rightarrow$ $\frac{-2-1}{7-(-2)}=\frac{3-2}{0-3}=\frac{4-3}{1-4}$

$\Rightarrow$ $\frac{-3}{9}=\frac{1}{-3}=\frac{1}{-3} \Rightarrow \frac{-1}{3}=\frac{-1}{3}=\frac{-1}{3}$

$\therefore$ Given points are collinear.

Here, $l = \cos\frac{\pi}{2} = 0$

$m = \cos\frac{3\pi}{4} = \cos\left(\pi-\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

and $n = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore$ Direction cosines are $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

Hence, both Assertion and Reason correct but Reason is not the correct explanation of Assertion.

**52. (c) :** In assertion the given cartesian equation is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$.

$\Rightarrow$ $\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$ and $\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$.

The vector equation of the line is given by $\vec{r}=\vec{a}+\lambda\vec{b}, \lambda\in R.$

$\Rightarrow$ $\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$

Thus Assertion is correct.

In reason it is given that the line passes through the point (–2, 4, –5) and is parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Clearly, the direction ratios of line are (3, 5, 6).

Now the equation of the line (in cartesian form) is

$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6} \Rightarrow \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Hence, Reason is wrong.

**53. (b) :** For first two lines, having direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ and $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$, we obtain

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{12}{13}\times\frac{4}{13}+\left(\frac{-3}{13}\right)\times\frac{12}{13}+\left(\frac{-4}{13}\right)\times\frac{3}{13}=0$$

Therefore, the lines are perpendicular.

For second and third lines having direction cosines $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ and $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$, we obtain

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{4}{13}\times\frac{3}{13}+\frac{12}{13}\times\left(\frac{-4}{13}\right)+\frac{3}{13}\times\frac{12}{13} = 0$$

Therefore, the lines are perpendicular.

For third and first lines, having directions cosines $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ and $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$, we obtain

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{3}{13}\times\frac{12}{13}+\left(\frac{-4}{13}\right)\times\left(\frac{-3}{13}\right)+\frac{12}{13}\times\left(\frac{-4}{13}\right)=0$$

Therefore, the lines are perpendicular.

Hence, all the lines are mutually perpendicular.

Let the given points are $A(1, -1, 2)$, $B(3, 4, -2)$, $C(0, 3, 2)$ and $D(3, 5, 6)$

Direction ratios of $AB$ are $(3 - 1, 4 - (-1), -2 - 2) = (2, 5, -4)$ and direction ratios of $CD$ are $(3 - 0, 5 - 3, 6 - 2) = (3, 2, 4)$

We know that, two lines $AB$ and $CD$ with direction ratios, $a_1, b_1, c_1$ and $a_2, b_2, c_2$ are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, $2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$

Therefore, the lines $AB$ and $CD$ are perpendicular.

Hence, both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**54. (a) :** Here, $\vec{a}_1 = \hat{i} - \hat{j}, \vec{b}_1 = 2\hat{i} + \hat{k}$

$\vec{a}_2 = 2\hat{i} - \hat{k}$ and $\vec{b}_2 = \hat{i} + \hat{j} - \hat{k}$

$\because$ $\vec{b}_1 \neq k\vec{b}_2$, for any scalar $k$

$\therefore$ Given lines are not parallel.

Now, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} - \hat{k}) - (\hat{i} - \hat{j}) = \hat{i} + \hat{j} - \hat{k}$

and $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 3\hat{j} + 2\hat{k}$

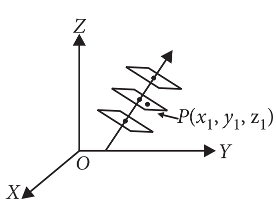
$\Rightarrow$ $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + (3)^2 + (2)^2} = \sqrt{1+9+4} = \sqrt{14}$

$$\therefore \text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \left|\frac{(\hat{i}+\hat{j}-\hat{k})\cdot(-\hat{i}+3\hat{j}+2\hat{k})}{\sqrt{14}}\right| = 0$$

Hence, two lines intersect each other.

Two lines intersect each other, if they are not parallel and shortest distance = 0.

**55. (d) :** In the space, there can be many planes that are perpendicular to the given vector, but if it also passes through a given point then only one such plane exist.




**56. (b) :** Given $\delta\theta$ be the angle between the two adjacent positions.

$\therefore$ $\cos\delta\theta = l(l + \delta l) + m(m + \delta m) + n(n + \delta n)$

$\Rightarrow$ $\cos\delta\theta = (l^2 + m^2 + n^2) + l\delta l + m\delta m + n\delta n$

$\Rightarrow$ $\cos\delta\theta = 1 + l\delta l + m\delta m + n\delta n$ ...(i)

Also, we have

$(l + \delta l)^2 + (m + \delta m)^2 + (n + \delta n)^2 = 1$

$\Rightarrow$ $(l^2 + m^2 + n^2) + ((\delta l)^2 + (\delta m)^2 + (\delta n)^{2,}$
$+ 2\,(l.\delta l + m.\delta m + n.\delta n) = 1$

$\Rightarrow$ $1 + ((\delta l)^2 + (\delta m)^2 + (\delta n)^2) + 2\,(\cos\delta\theta - 1) = 1$ [using (i)]

$\Rightarrow$ $(\delta l)^2 + (\delta m)^2 + (\delta n)^2 = 2\,(1 - \cos\delta\theta)$

$= 2\left(2.\sin^2\left(\frac{\delta\theta}{2}\right)\right) = 4\left(\sin\left(\frac{\delta\theta}{2}\right)\right)^2$

$= 4\left(\frac{\delta\theta}{2}\right)^2$ [$\because$ For very small angle $\theta$, $\sin\theta = \theta$]

$= (\delta\theta)^2$

$\Rightarrow$ $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$

In reason, we have $\vec{n} = \overrightarrow{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

The required equation of plane is given by

$[\vec{r} - (a\hat{i} + b\hat{j} + c\hat{k})]\cdot\vec{n} = 0$

$\Rightarrow$ $\vec{r}\cdot(a\hat{i}+b\hat{j}+c\hat{k}) - (a\hat{i}+b\hat{j}+c\hat{k})\cdot(a\hat{i}+b\hat{j}+c\hat{k}) = 0$

$\Rightarrow$ $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(a\hat{i}+b\hat{j}+c\hat{k}) - (a^2+b^2+c^2) = 0$

$\Rightarrow$ $ax + by + cz = a^2 + b^2 + c^2$

**57. (a) :** Lines $L_1$ and $L_2$ are parallel to the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ respectively. The unit vector perpendicular to both $L_1$ and $L_2$ is

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt{1+49+25}} = \frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$$

Now, equation of plane through $(-1, -2, -1)$ is $-(x + 1) - 7(y + 2) + 5(z + 1) = 0$ whose distance from $(1, 1, 1)$ is $\frac{13}{5\sqrt{3}}$.

**58. (c) :** Here, $\vec{N} = (2-3)\hat{i} + (-1-4)\hat{j} + (5+1)\hat{k}$
$= -\hat{i} - 5\hat{j} + 6\hat{k}$

Now, equation of the plane passing through $(2, -3, 1)$ perpendicular to $\vec{N}$ is

$-1(x - 2) - 5(y + 3) + 6(z - 1) = 0$

$\Rightarrow$ $-x + 2 - 5y - 15 + 6z - 6 = 0 \Rightarrow x + 5y - 6z + 19 = 0$

which is the required equation.

$$\text{Length of perpendicular} = \left|\frac{2(7)+4(14)-(5)-2}{\sqrt{2^2+4^2+(-1)^2}}\right| = 3\sqrt{21}$$

**59. (b) :** Equation of the plane which cuts the axes at distances $a, b, c$ is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Since, the same plane passes through $(a', 0, 0)$, $(0, b', 0)$ and $(0, 0, c')$ in other system.

Therefore, $\frac{a'}{a} = 1 \Rightarrow \frac{1}{a} = \frac{1}{a'}$

$\frac{b'}{b} = 1 \Rightarrow \frac{1}{b} = \frac{1}{b'}$ and $\frac{c'}{c} = 1 \Rightarrow \frac{1}{c} = \frac{1}{c'}$

$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{(a')^2} + \frac{1}{(b')^2} + \frac{1}{(c')^2}$

Let $d_1$ and $d_2$ be the distances of the point $(\hat{i} - \hat{j} + 3\hat{k})$ and $(3\hat{i} + 3\hat{j} + 3\hat{k})$ from the plane.

Then, $d_1 = \left| \frac{(\hat{i} - \hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9}{\sqrt{5^2 + 2^2 + (-7)^2}} \right| = \frac{9}{\sqrt{78}}$

$d_2 = \left| \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9}{\sqrt{5^2 + 2^2 + (-7)^2}} \right| = \frac{9}{\sqrt{78}}$

Clearly, $d_1 = d_2$

**60. (a) :** Any point on the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{13}$ is (3, 4, –3), which satisfies the equation of the plane $5x - y + z = 8$.

Also, direction cosines of the line are

$< \frac{-4}{\sqrt{234}}, \frac{-7}{\sqrt{234}}, \frac{13}{\sqrt{234}} >$ and direction cosines of the normal to the plane $5x - y + z = 8$ are < 5, –1, 1 >.

$\therefore \quad al + bm + cn = \frac{-20}{\sqrt{234}} + \frac{7}{\sqrt{234}} + \frac{13}{\sqrt{234}} = 0$.

## SUBJECTIVE TYPE QUESTIONS

**1.** The eq. of given line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ ...(i)

$\Rightarrow \quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} \quad \Rightarrow \quad \frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$

As $\sqrt{2^2 + (-6)^2 + 3^2} = 7$

$\therefore$ D.c's. of (i) are $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$.

**2.** Vector equation of a line passes through the points (3, 4, –7) and (1, –1, 6) is given by

$\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda[(\hat{i} - \hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} - 7\hat{k})]$

$\therefore \quad \vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

**3.** The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$\Rightarrow \quad \frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

*i.e.*, Its direction ratios are proportional to 6, 2, –3.

Now, $\sqrt{6^2 + 2^2 + (-3)^2} = 7$

$\therefore$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

**4.** We have, $2x + y - z = 5$

$\Rightarrow \quad \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

which is the equation of plane in intercept form.

$\therefore$ Intercepts on $x$, $y$ and $z$-axis respectively are $\frac{5}{2}, 5, -5$.

$\therefore$ Required length of intercept $= \frac{5}{2}$

**5.** Vector eq. of the line passing through (1, –1, 2) and parallel to the line, $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ is

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$.

**6.** Let $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore \quad \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4+9+36}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$

So, the required equation of the plane is

$\vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 5 \quad \Rightarrow \quad \vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$

**7.** Let $\alpha$, $\beta$ and $\gamma$ be the angles made by $\vec{n}$ with $x$, $y$ and $z$-axis, respectively.

Given $\alpha = \beta = \gamma \Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow l = m = n$, where $l, m, n$ are direction cosines of $\vec{n}$.

But $l^2 + m^2 + n^2 = 1 \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

So, $l = m = n = \pm\frac{1}{\sqrt{3}}$

$\therefore$ The normal form of the plane is $lx + my + nz = d$

$\Rightarrow \quad \pm\frac{1}{\sqrt{3}}x \pm \frac{1}{\sqrt{3}}y \pm \frac{1}{\sqrt{3}}z = 5\sqrt{3}$

$\Rightarrow \quad \pm x \pm y \pm z = 15$

**8.** Equation of both the planes can be written as $2x - y + 2z = 5$ and $2x - y + 2z = 8$.

Distance between both the planes

$= \left| \frac{8-5}{\sqrt{4+1+4}} \right| = \frac{3}{\sqrt{9}} = 1$ unit

**9.** Perpendicular distance from the origin (0, 0, 0) to the plane $3x - 4y + 12z - 3 = 0$ is

$\left| \frac{3 \times 0 - 4 \times 0 + 12 \times 0 - 3}{\sqrt{3^2 + (-4)^2 + 12^2}} \right| = \frac{3}{13}$ unit.

**10.** The given plane is $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

$\Rightarrow \quad \vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

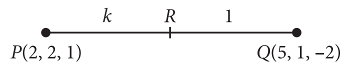
$\because$ D.r's of $\perp$ to this plane are 1, 2, –5.

So, the line has direction ratios proportional to 1, 2, –5.

$\therefore$ Eq. of line through (1, 2, 3) and $\perp$ to the plane is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$.

**11.** Given that $P(2, 2, 1)$ and $Q(5, 1, -2)$




Let the point $R$ on the line $PQ$, divides the line in the ratio $k : 1$ and $x$-coordinate of point $R$ on the line is 4.
So, by section formula

$$4 = \frac{5k+2}{k+1} \Rightarrow k = 2$$

Now, $z$-coordinate of point $R$,

$$z = \frac{-2k+1}{k+1} = \frac{-2\times 2+1}{2+1} = -1$$

$\Rightarrow$ $z$-coordinate of point $R = -1$

**12.** The given lines are

$$l_1 : \frac{x-0}{1} = \frac{y-0}{-1} = \frac{z-0}{\frac{1}{k}}$$

$$l_2 : \frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$$

$\because$ $l_1$ is perpendicular to $l_2$.

$$\therefore \quad 1(1)+(-1)\left(\frac{1}{2}\right)+\left(\frac{1}{k}\right)(-1)=0$$

$$\Rightarrow \quad 1-\frac{1}{2}-\frac{1}{k}=0 \Rightarrow \frac{1}{2}=\frac{1}{k} \Rightarrow k=2$$

**13.** Vector equation of the line passing through $(1, 2, -1)$ and parallel to the line
$5x - 25 = 14 - 7y = 35z$

$$i.e., \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35} \text{ or } \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$$

is $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

**14.** The equation of line through $A(-1, 1, -8)$ and $B(5, -2, 10)$ is $\frac{x+1}{5+1} = \frac{y-1}{-2-1} = \frac{z+8}{10+8}$

$$i.e., \frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = k(\text{say}) \quad \text{...(i)}$$

Any point on (i) is given by $(6k - 1, -3k + 1, 18k - 8)$.
We know that the coordinates of any point on the $ZX$-plane is given by $(x, 0, z)$.

$$\therefore \quad -3k + 1 = 0 \Rightarrow k = \frac{1}{3}$$

Thus the coordinates of the point where the line joining $A$ and $B$ crosses the $ZX$-plane are

$$\left(6\times\frac{1}{3}-1, 0, 18\times\frac{1}{3}-8\right) = (1, 0, -2)$$

**15.** Any point on the line

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r \text{ (say)} \quad \text{...(i)}$$

is $(3r - 1, 5r - 3, 7r - 5)$.
Any point on the line

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = k \text{ (say)} \quad \text{...(ii)}$$

is $(k + 2, 3k + 4, 5k + 6)$
For lines (i) and (ii) to intersect, we must have
$3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$
On solving these, we get $r = \frac{1}{2}, k = -\frac{3}{2}$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

**16.** Any point on the given line,

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = k \text{ (say)} \quad \text{...(i)}$$

is $R(2k + 1, -3k - 1, 8k - 10)$
If this is the foot of the $\perp$ from $P(1, 0, 0)$ on (i), then
$(2k + 1 - 1) \cdot 2 + (-3k - 1 - 0) \cdot (-3) + (8k - 10 - 0) \cdot 8 = 0$
$\Rightarrow$ $4k + 9k + 3 + 64k - 80 = 0$
$\Rightarrow$ $77k = 77 \Rightarrow k = 1$
$\therefore$ $R$ is $(3, -4, -2)$.
This is the required foot of perpendicular.
Also, perpendicular distance $= PR$

$$= \sqrt{(3-1)^2+(-4-0)^2+(-2-0)^2} = \sqrt{24} = 2\sqrt{6} \text{ units.}$$

Also eq. of $PR$ is $\frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2}$

**17.** The given lines are

$$l_1 : \frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2}$$

and $l_2 : \frac{x-1}{-3\lambda/7} = \frac{y-5}{1} = \frac{z-6}{-5}$

Now, $l_1 \perp l_2$ [Given]

$$\therefore \quad (-3)\left(-\frac{3\lambda}{7}\right)+\frac{\lambda}{7}-10=0$$

$$\Rightarrow \quad \frac{9\lambda}{7}+\frac{\lambda}{7}-10=0 \Rightarrow \frac{10\lambda}{7}=10 \Rightarrow \lambda=7$$

Since for $\lambda = 7$, given lines are at right angles.
$\therefore$ Lines are intersecting.

**18.** Let $l, m, n$ be the direction ratios of the line which is perpendicular to the lines
$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.
Then, $l + 2m + 3n = 0$ and $-3l + 2m + 5n = 0$

$$\Rightarrow \quad \frac{l}{10-6} = \frac{m}{-9-5} = \frac{n}{2+6} \Rightarrow \frac{l}{2} = \frac{m}{-7} = \frac{n}{4}$$

$\therefore$ Eq. of the required line through $(-1, 3, -2)$ having d.r's proportional to $2, -7, 4$ is

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}.$$

**19.** Equation of given line is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = k(\text{say})$$

$\Rightarrow$ $x = 3k + 2, y = 4k - 1, z = 12k + 2$
Since point $(3k + 2, 4k - 1, 12k + 2)$ lie on plane
$x - y + z = 5$

$\therefore$ $3k + 2 - 4k + 1 + 12k + 2 = 5$

$\Rightarrow$ $11k = 0 \Rightarrow k = 0$

$\therefore$ Point is $(2, -1, 2)$

Required distance $= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$

$= \sqrt{9+16+144} = \sqrt{169} = 13$ units

**20.** Given that $-6, 3, 4$ are intercepts on $x$, $y$ and $z$-axes respectively.

$\therefore$ Eq. of the plane is $\frac{x}{-6} + \frac{y}{3} + \frac{z}{4} = 1$

$\Rightarrow$ $-2x + 4y + 3z - 12 = 0$

$\therefore$ Length of the perpendicular from origin to the plane $-2x + 4y + 3z - 12 = 0$ is

$$\left|\frac{-2\times 0 + 4\times 0 + 3\times 0 - 12}{\sqrt{(-2)^2 + 4^2 + 3^2}}\right| = \frac{12}{\sqrt{29}} \text{ unit}$$

**21.** The equation of line $AB$ is given by

$$\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{1+1} = \lambda \text{ (say)}$$

$\Rightarrow$ $x = 4\lambda$, $y = 6\lambda - 1$, $z = 2\lambda - 1$

The coordinates of a general point on $AB$ are $(4\lambda, 6\lambda - 1, 2\lambda - 1)$

The equation of line $CD$ is given by

$$\frac{x-3}{3+4} = \frac{y-9}{9-4} = \frac{z-4}{4-4} = \mu \text{ (say)}$$

$\Rightarrow$ $x = 7\mu + 3$, $y = 5\mu + 9$, $z = 4$

The coordinates of a general point on $CD$ are $(7\mu + 3, 5\mu + 9, 4)$

If the line $AB$ and $CD$ intersect then they have a common point. So, for some values of $\lambda$ and $\mu$, we must have

$4\lambda = 7\mu + 3$, $6\lambda - 1 = 5\mu + 9$, $2\lambda - 1 = 4$

$\Rightarrow$ $4\lambda - 7\mu = 3$ ...(i), $6\lambda - 5\mu = 10$ ...(ii)

and $\lambda = \frac{5}{2}$ ...(iii)

Substituting $\lambda = \frac{5}{2}$ in (ii), we get $\mu = 1$

Since $\lambda = \frac{5}{2}$ and $\mu = 1$ satisfy (i), so the given lines $AB$ and $CD$ intersect.

**22.** The given lines are

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$\Rightarrow$ $\vec{r} = (3+\lambda)\hat{i} + (2+2\lambda)\hat{j} + (2\lambda - 4)\hat{k}$ ...(i)

and $\vec{r} = (5+3\mu)\hat{i} + (2\mu - 2)\hat{j} + 6\mu\hat{k}$ ...(ii)

If these lines intersect, they must have a common point. So, we must have

$(3+\lambda)\hat{i} + (2+2\lambda)\hat{j} + (2\lambda - 4)\hat{k} = (5+3\mu)\hat{i} + (2\mu - 2)\hat{j} + 6\mu\hat{k}$

$\Rightarrow$ $3 + \lambda = 5 + 3\mu \Rightarrow \lambda - 3\mu = 2$,

$2 + 2\lambda = 2\mu - 2 \Rightarrow \lambda - \mu = -2$,

and $2\lambda - 4 = 6\mu \Rightarrow \lambda - 3\mu = 2$

$\Rightarrow$ $\lambda = -4$, $\mu = -2$.

$\therefore$ The given lines intersect and their point of intersection is $(-1, -6, -12)$.

**23.** Here $\vec{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{b} = 7\hat{i} - \hat{k}$

$\therefore$ Equation of line joining $A$ and $B$ is,

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\Rightarrow$ $\vec{r} = -2\hat{i} + 3\hat{j} + 5\hat{k} + \lambda(9\hat{i} - 3\hat{j} - 6\hat{k})$ ... (i)

Again, $\vec{c} = -3\hat{i} - 2\hat{j} - 5\hat{k}$, $\vec{d} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$\therefore$ Equation of line joining $C$ and $D$ is,

$\vec{r} = \vec{c} + \mu(\vec{d} - \vec{c})$

$\Rightarrow$ $\vec{r} = -3\hat{i} - 2\hat{j} - 5\hat{k} + \mu(6\hat{i} + 6\hat{j} + 12\hat{k})$ ... (ii)

Equations (i) and (ii) will intersect, when

$-2\hat{i} + 3\hat{j} + 5\hat{k} + \lambda(9\hat{i} - 3\hat{j} - 6\hat{k}) = -3\hat{i} - 2\hat{j} - 5\hat{k} + \mu(6\hat{i} + 6\hat{j} + 12\hat{k})$

$\Rightarrow$ $-2 + 9\lambda = -3 + 6\mu$ ; $3 - 3\lambda = -2 + 6\mu$ ; $5 - 6\lambda = -5 + 12\mu$

$\Rightarrow$ $9\lambda - 6\mu = -1$ ; $3\lambda + 6\mu = 5$ ; $6\lambda + 12\mu = 10$

Solving first two equations, we get $\lambda = \frac{1}{3}, \mu = \frac{2}{3}$ which also satisfy third equation.

Put $\lambda = \frac{1}{3}$ in (i), we get the point of intersection of lines is $\hat{i} + 2\hat{j} + 3\hat{k} \equiv (1, 2, 3)$.

**24.** The given lines are

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Equation of any line through $(1, 2, -4)$ with d.r's $l, m, n$ is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(l\hat{i} + m\hat{j} + n\hat{k})$ ...(i)

Since, the required line is perpendicular to both the given lines.

$\therefore$ $3l - 16m + 7n = 0$ and $3l + 8m - 5n = 0$

$\Rightarrow$ $\frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48} \Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$

$\therefore$ From (i), the required line is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here, the position vector of passing point is $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$ and parallel vector is $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore$ Cartesian equation is given by

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

**25.** Here, the lines are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Here, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$ and

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

The shortest distance between the lines is given by

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$\Rightarrow \ |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + 3^2 + 9^2} = 3\sqrt{19}$

Also, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)$

$= (3\hat{i} + 3\hat{j} + 3\hat{k})\cdot(-9\hat{i} + 3\hat{j} + 9\hat{k})$

$= 3 \times (-9) + 3 \times 3 + 3 \times 9 = 9$

$\therefore \ d = \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$ unit.

**26.** The given line is $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$

$\Rightarrow \ \frac{x+2}{2} = \frac{y - \frac{7}{2}}{3} = \frac{z-5}{-6}$ ...(i)

Its d.r's are 2, 3, –6

$\because \ \sqrt{2^2 + 3^2 + (-6)^2} = 7$

$\therefore$ Its d.c's are $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Eq. of a line through (–1, 2, 3) and parallel to (i) is

$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-3}{-6} = \lambda$ (say)

$\therefore$ Vector equation of a line passing through (–1, 2, 3) and parallel to (i) is given by

$\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$

**27.** The given lines are

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

S.D. between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

On comparing, we get

$\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}$, $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{a}_2 = 7\hat{i} - 6\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \ \vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 6 \\ 1 & 2 & 2 \end{vmatrix} = -8\hat{i} + 4\hat{k}$$

$\therefore \ |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-8)^2 + 4^2} = 4\sqrt{5}$

Hence, $d = \left| \frac{(5\hat{i} + 5\hat{j} - 7\hat{k})\cdot(-8\hat{i} + 4\hat{k})}{4\sqrt{5}} \right|$

$= \frac{|5(-8) - 7(4)|}{4\sqrt{5}} = \frac{68}{4\sqrt{5}} = \frac{17\sqrt{5}}{5}$ units

**28.** Let $l_1 : \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

$\Rightarrow \ \frac{x-(-1)}{7} = \frac{y-(-1)}{-6} = \frac{z-(-1)}{1}$

and $l_2 : \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

$\therefore$ Vector equation of lines are

$\vec{r} = -\hat{i} - \hat{j} - \hat{k} + \lambda(7\hat{i} - 6\hat{j} + \hat{k})$

and $\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \mu(\hat{i} - 2\hat{j} + \hat{k})$

We get $\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$, $\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

So, $\vec{a}_2 - \vec{a}_1 = (3\hat{i} + 5\hat{j} + 7\hat{k}) - (-\hat{i} - \hat{j} - \hat{k}) = 4\hat{i} + 6\hat{j} + 8\hat{k}$

And, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

Shortest distance between two skew lines is,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\Rightarrow \ d = \left| \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}} \right|$

$\Rightarrow \ d = \left| \frac{-16 - 36 - 64}{\sqrt{116}} \right| \ \Rightarrow \ d = 2\sqrt{29}$ units

**29.** Here the position vectors of $A$, $B$ and $C$ are respectively $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$.

$\therefore \ \overrightarrow{AB} = (\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{AC} = (2\hat{i} + 3\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = 0 \cdot \hat{i} + \hat{j} + 2\hat{k}$

A vector normal to the plane containing points $A$, $B$ and $C$ is

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ The required unit vector

$$= \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{3^2 + 2^2 + (-1)^2}} = \frac{1}{\sqrt{14}}(3\hat{i} + 2\hat{j} - \hat{k})$$

**30.** Let the eq. of the plane through (2, 1, –1) be

$a(x - 2) + b(y - 1) + c(z + 1) = 0$ ...(i)

Also, point (–1, 3, 4) lies on it.

$\therefore \ -3a + 2b + 5c = 0$ ...(ii)

Also (i) is $\perp$ to the plane $x - 2y + 4z = 10$

$\therefore \ a \cdot 1 + b(-2) + c \cdot 4 = 0$

$\Rightarrow \quad a - 2b + 4c = 0$ ...(iii)

Solving (ii) and (iii), we get

$$\frac{a}{8+10} = \frac{b}{5+12} = \frac{c}{6-2} \quad \Rightarrow \quad \frac{a}{18} = \frac{b}{17} = \frac{c}{4}$$

$\therefore$ From (i), required eqn. of the plane is

$18(x - 2) + 17(y - 1) + 4(z + 1) = 0$

$\Rightarrow \quad 18x + 17y + 4z = 49$

In vector form, eq. of this plane is

$\vec{r} \cdot (18\hat{i} + 17\hat{j} + 4\hat{k}) = 49.$

**31.** The given lines are

$$\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$$

$$\Rightarrow \quad \frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

The condition of coplanarity of these lines is

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\text{Here, L.H.S.} = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} 8-5 & 4-7 & 5-(-3) \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$= 3(12 + 5) + 3(12 + 35) + 8(4 - 28)$

$= 51 + 141 - 192 = 0 =$ R.H.S.

Hence the given lines are coplanar.

**32.** Let $P$ be the point with position vector

$\hat{i} + 3\hat{j} + 4\hat{k} \equiv (1, 3, 4)$ and the plane is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

$\Rightarrow 2x - y + z + 3 = 0$ ...(i)

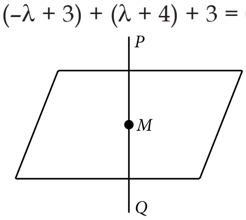
Any line $\perp$ to this plane passes through $P(1, 3, 4)$ is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \text{ (say)}$$

Any point $M$ on the line is $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

This lies on the plane (i).

$\therefore \quad 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$




$\Rightarrow \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1.$

$\therefore$ Coordinates of point $M$ are $(-1, 4, 3)$.

Let the image of $P$ in the plane (i) be $Q(\alpha, \beta, \gamma)$, then $M$ will be the midpoint of $PQ$.

$$\therefore \quad \frac{\alpha+1}{2} = -1, \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3$$

$\Rightarrow \quad \alpha = -3, \beta = 5, \gamma = 2$

Hence, $Q(-3, 5, 2)$ is the image of $P(1, 3, 4)$ in plane (i).

**33.** Eq. of any plane through $(-1, 2, 0)$ is

$a(x + 1) + b(y - 2) + cz = 0$ ...(i)

Also it passes through $(2, 2, -1)$

$\therefore \quad 3a + 0 \cdot b - c = 0$ ...(ii)

Further plane (i) is parallel to the line

$$\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$$

$\therefore \quad a \cdot 1 + b \cdot 1 + c \cdot (-1) = 0$

$\Rightarrow \quad a + b - c = 0$ ...(iii)

Eliminating $a$, $b$ and $c$ from (ii) and (iii), we get

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda\text{(say)}$$

$\Rightarrow \quad a = \lambda, b = 2\lambda$ and $c = 3\lambda$

Put these values in (i), we get

$x + 2y + 3z = 3$

This is the eq. of the required plane.

**34.** Equation of plane passing through $(3, 2, 0)$ is

$a(x - 3) + b(y - 2) + c(z - 0) = 0$ ...(i)

Given line is $\dfrac{x-3}{1} = \dfrac{y-6}{5} = \dfrac{z-4}{4}$

Since plane contains the line, so

$a(3 - 3) + b(6 - 2) + c(4 - 0) = 0$

$\Rightarrow \quad 0 \cdot a + 4b + 4c = 0$ ...(ii)

and $a(1) + b(5) + c(4) = 0$

$\Rightarrow \quad a + 5b + 4c = 0$ ...(iii)

Solving (ii) & (iii), we get

$$\frac{a}{16-20} = \frac{b}{4-0} = \frac{c}{0-4} = \lambda \text{ (say)}$$

$\Rightarrow \quad a = -4\lambda, b = 4\lambda, c = -4\lambda$

Putting values of $a$, $b$, $c$ in (i), we get

$-4\lambda(x - 3) + 4\lambda(y - 2) - 4\lambda(z - 0) = 0$

$\Rightarrow \quad -4x + 12 + 4y - 8 - 4z = 0$

$\Rightarrow \quad x - y + z - 1 = 0$ is the required equation of plane.

**35.** The eq. of the line through $A(3, 4, 1)$ and

$B(5, 1, 6)$ is $\dfrac{x-3}{5-3} = \dfrac{y-4}{1-4} = \dfrac{z-1}{6-1}$

$$\Rightarrow \quad \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} \quad \text{...(i)}$$

Now (i) meets the $XY$-plane (whose eq. is $z = 0$)

$$\therefore \quad \frac{x-3}{2} = \frac{y-4}{-3} = \frac{0-1}{5} = -\frac{1}{5}$$

$$\Rightarrow \quad \frac{x-3}{2} = -\frac{1}{5}, \frac{y-4}{-3} = -\frac{1}{5}, z = 0$$

$$\Rightarrow \quad x = 3 - \frac{2}{5}, y = 4 + \frac{3}{5}, z = 0$$

$$\Rightarrow \quad x = \frac{13}{5}, y = \frac{23}{5}, z = 0$$

$\Rightarrow \left(\dfrac{13}{5}, \dfrac{23}{5}, 0\right)$ is the point where line (i) meets the $XY$-plane.

**36.** The given line is

$\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

Its cartesian eq. is

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda(\text{say}) \quad ...(i)$$

Any point $Q$ on (i) is $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

Also, the given point is $P(5, 4, 2)$.

Now d.r's of the line $PQ$ are

$(2\lambda - 1 - 5, 3\lambda + 3 - 4, -\lambda + 1 - 2) = (2\lambda - 6, 3\lambda - 1, -\lambda - 1)$.

For $PQ$ to be $\perp$ to (i), we must have

$(2\lambda - 6)\cdot 2 + (3\lambda - 1)\cdot 3 + (-\lambda - 1)\cdot(-1) = 0$

$\Rightarrow \quad 14\lambda - 14 = 0 \Rightarrow \lambda = 1$

$\therefore \quad Q$ is $(1, 6, 0)$

which is the foot of $\perp$ from $P$ on line (i).

Now, $PQ = \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$

$= \sqrt{24} = 2\sqrt{6}$ units.

Further if $R(\alpha, \beta, \gamma)$ is the image of $P$ in line (i), then

$$\frac{\alpha+5}{2} = 1, \frac{\beta+4}{2} = 6, \frac{\gamma+2}{2} = 0$$

$\Rightarrow \quad \alpha = -3, \beta = 8, \gamma = -2$

$\therefore$ Image of $P$ in line (i) is $R(-3, 8, -2)$.

**37.** The given line is

$\vec{r} = (2\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$

$$\Rightarrow \frac{x-2}{3} = \frac{y+4}{4} = \frac{z-2}{2} = \lambda \text{ (say)} \quad ...(i)$$

Any point on it is $(3\lambda + 2, 4\lambda - 4, 2\lambda + 2)$

This lies on the plane $\vec{r}\cdot(\hat{i} - 2\hat{j} + \hat{k}) = 0$

$\Rightarrow \quad x - 2y + z = 0 \quad ...(ii)$

$\therefore \quad 3\lambda + 2 - 2(4\lambda - 4) + 2\lambda + 2 = 0$

$\Rightarrow \quad -3\lambda + 12 = 0 \Rightarrow \lambda = 4$

$\therefore$ The point of intersection of (i) and (ii) is

$(3 \times 4 + 2, 4 \times 4 - 4, 2 \times 4 + 2) = (14, 12, 10)$

Its distance from the point $(2, 12, 5)$

$= \sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2}$

$= \sqrt{144 + 0 + 25} = \sqrt{169} = 13$ units.

**38.** Vector equation of a line passing through $(2, 3, 2)$ and parallel to the line

$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ is given by

$\vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

Now, $\vec{a}_1 = -2\hat{i} + 3\hat{j}, \ \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Distance between given parallel lines

$$= \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right| = \left|\frac{(2\hat{i} - 3\hat{j} + 6\hat{k})\cdot(4\hat{i} + 0\hat{j} + 2\hat{k})}{|\sqrt{4+9+36}|}\right|$$

$$= \left|\frac{8+0+12}{\sqrt{49}}\right| = \frac{20}{7} \text{ units}$$

**39.** Any point on the line

$$\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1} = r(\text{say}) \quad ...(i)$$

is $(r + 2, 3r + 2, r + 3)$

Any point on the line

$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2} = k(\text{say}) \quad ...(ii)$$

is $(k + 2, 4k + 3, 2k + 4)$

For lines (i) and (ii) to intersect, we must have

$r + 2 = k + 2, 3r + 2 = 4k + 3, r + 3 = 2k + 4$

On solving these, we get, $r = k = -1$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $(1, -1, 2)$

Now, required equation of plane is

$$\begin{vmatrix} x-2 & y-2 & z-3 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad (x - 2)(6 - 4) - (y - 2)(2 - 1) + (z - 3)(4 - 3) = 0$

$\Rightarrow \quad 2x - 4 - y + 2 + z - 3 = 0 \Rightarrow \quad 2x - y + z - 5 = 0$

**40.** The given lines are

$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \quad ...(i)$

and $\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k}) \quad ...(ii)$

Here, $\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b}_1 = \hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} + 2\hat{k}, \vec{b}_2 = 3\hat{i} - 2\hat{j} + 5\hat{k}$

The plane containing lines (i) and (ii) will pass through $\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}$.

Also the plane is parallel to two vectors $\vec{b}_1$ and $\vec{b}_2$.

$\therefore$ The plane is normal to the vector

$$\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{i} + 10\hat{j} - 8\hat{k}$$

$\therefore$ The vector eqn. of the required plane is

$(\vec{r} - \vec{a}_1)\cdot\vec{n} = 0 \Rightarrow \vec{r}\cdot\vec{n} = \vec{a}_1\cdot\vec{n}$

$\Rightarrow \vec{r}\cdot(20\hat{i} + 10\hat{j} - 8\hat{k}) = (2\hat{i} + \hat{j} - 3\hat{k})\cdot(20\hat{i} + 10\hat{j} - 8\hat{k})$

$= 40 + 10 + 24 = 74$

$\Rightarrow \quad \vec{r}\cdot(10\hat{i} + 5\hat{j} - 4\hat{k}) = 37 \quad ...(iii)$

Its cartesian eqn. is $10x + 5y - 4z = 37$

The given line is,

$\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + p(3\hat{i} - 2\hat{j} + 5\hat{k}) \quad ...(iv)$

The line (iv) will lie in the plane (iii) if the plane passes through the point $\vec{a} = 2\hat{i} + 5\hat{j} + 2\hat{k}$ on line (iv) and is parallel to the line (iv).

Now, $\vec{a}\cdot(10\hat{i} + 5\hat{j} - 4\hat{k})$

$= (2\hat{i} + 5\hat{j} + 2\hat{k})\cdot(10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$

$\therefore \quad \vec{a}$ lies in the plane (iii)

Also, $(10\hat{i} + 5\hat{j} - 4\hat{k})\cdot(3\hat{i} - 2\hat{j} + 5\hat{k})$

$= 10 \times 3 + 5 \times (-2) - 4 \times 5 = 0$

$\therefore$ Line (iv) is parallel to the plane (iii).

$\therefore$ Line (iv) lies in the plane (iii).